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Scaling properties of the localization length in one-dimensional paired correlated binary alloys of finite size

Felix M Izrailev†‡‖, Tsampikos Kottos†§ and G P Tsironis†

† Department of Physics, University of Crete and Research Centre of Crete, PO Box 1527, 71110 Heraklion, Crete, Greece

‡ Budker Institute of Nuclear Physics, Novosibirsk 630090, Russia

§ Istituto Nazionale di Ottica, 50125 Firenze, Italy

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Abstract. We study scaling properties of the localized eigenstates of the random dimer model in which pairs of local site energies are assigned at random in a one-dimensional disordered tight-binding model. We use both the transfer matrix method and the direct diagonalization of the Hamiltonian in order to find how the localization length of a finite sample scales with the localization length of the infinite system. We derive the scaling law for the localization length and show it to be related to scaling behaviour typical of uncorrelated band random matrix, Anderson and Lloyd models.

1. Introduction

It is well known that in one-dimensional (1D) disordered models even small amounts of disorder lead to an exponential localization of all eigenstates [1, 2]. On the other hand, recent studies of quasi-1D polymers have shown that short-range correlations embedded in a random sequence can lead to the appearance of fully transparent states [3, 4]. In [4] in particular, various organic disordered systems with electrical properties were quoted. The prototypical case is that of the random dimer model (RDM) [3, 4] where (in the context of a tight-binding Hamiltonian) pairs of adjacent energy levels are assigned at random, leading to two-site correlations in an otherwise random model. Since for infinite samples fully delocalized states appear only for specific energy values E_{cr}, there is no Anderson transition in the usual sense (see also [5]). However, the number of transparent states for finite samples was found to be proportional to the square root of the length of the sample [3, 4]. This fact is related to the divergence of the localization length in infinite samples when the energy approaches some critical values [6, 7, 8]. Therefore, these states may be important for conducting properties of finite samples [9, 10].

In infinite samples the Anderson transition can be characterized in terms of the localization length; the latter is commonly defined from the decay of amplitude of eigenstates in the limit $|n| \to \infty$, where n is the site label in the tight-binding picture. Contrary to what happens in infinite samples, the global properties of eigenstates of finite samples cannot be characterized in the same way; one needs to use other quantities (such as the inverse

‖ E-mail addresses: izrailev@vxinpb.inp.nsk.su; izrailev@physics.spa.umn.edu.

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participation ratio), that are valid for both finite and infinite samples. Then, through the use of scaling conjectures, one can link the properties of eigenstates in infinite samples to those in finite samples. In the theory of disordered solids, the scaling approach proved to be extremely useful in describing the conductance and its fluctuations (see, e.g., [11, 12]). A similar approach has been recently used in the theory of quantum chaos when describing localized eigenstates random on a finite scale [13, 14]. Such eigenstates also arise in the quasi-1D models with random potentials. Extensive numerical and analytical studies (see, e.g., [15, 16] and references therein) have revealed remarkable scaling properties of eigenstates, which seem to be of quite generic nature.

In this paper we study the RDM of finite size from the point of view of scaling properties of its eigenstates. The question of the relevance of the above-mentioned results to models with correlated disorder is far from being trivial since short-range correlations may cause significant difference in the structure of eigenstates, when compared with those for random potentials. In the next section we briefly describe the RDM and discuss different definitions of localization length, which are used in our numerical simulations. In section 3 we present numerical data on scaling properties of eigenstates in the centres of energy bands. In this case, the localization length in infinite samples has been obtained by the transfer matrix method. In section 4, we study the energy region near the critical values E_{cr} by making use of both numerical and analytical treatment of the localization length. Finally, in section 5 we give a short summary of our investigation.

2. A finite-size scaling approach to the random dimer model

Our starting point is the 1D Schrödinger equation in the tight-binding approximation

$$\mathrm{i}\frac{\mathrm{d}c_n(t)}{\mathrm{d}t} = \epsilon_n c_n(t) + c_{n+1}(t) + c_{n-1}(t) \tag{1}$$

where $c_n(t)$ is the probability amplitude for an electron to be at site n and ϵ_n is the local site energy. By making the transformation $c_n(t) = \exp(-\mathrm{i}Et)\phi_n$ one can obtain the equation

$$E\varphi_n = \varphi_{n+1} + \epsilon_n\varphi_n + \varphi_{n-1} \tag{2}$$

for the eigenvalue E and the corresponding eigenstate $\varphi_n(E)$. In what follows, we consider the RDM [3, 8, 17] which implies short-range correlations in the sequence of random ϵ_n. In this model there are only two values of ϵ_n, namely ϵ_A or ϵ_B, that appear in pairs in the sequence of ϵ_n's. In other words, in order to create the dimer chain, the pairs AA and BB each with energies ϵ_A, ϵ_B respectively are distributed at random. We take for simplicity the probabilities of occurrence of the pairs to be equal, i.e. $\mathcal{P}_{AA} = \mathcal{P}_{BB} = 1/2$.

The RDM has been well studied for an infinite chain (see, e.g., [3, 5, 6, 7, 8, 17]). Its basic feature is that for two values of the energy $E_{cr} = \epsilon_A$ or ϵ_B its eigenstates are extended. In the vicinity of these energies and for $\epsilon_{A,B} = |\epsilon_A - \epsilon_B|$ less than the critical value $\epsilon^{cr}_{A,B} = 2$, the localization length l_∞ defined through the exponential decay of the amplitude of the eigenfunction diverges as $l_\infty(E) \sim 1/E^2$ (for the specific value $\epsilon_{A,B} = 2$, the singularity law $l_\infty(E) \sim 1/E$ holds instead; see details in [6, 7, 8]). In spite of the fact that for other values of E inside the spectrum the localization length is finite, the influence of nearly transparent states on the electronic properties of finite samples is strong (see, e.g., [9, 10]). This is related to the fact that the number of eigenstates with localization length larger than the size N of the sample is proportional to $\sqrt{N}$.

Our main interest is in the structure of the eigenstates for finite samples, both in the centre of the energy band and in the vicinity of the critical energies E_{cr} where the localization

length for the infinite sample diverges. Unlike in the simpler case of infinite samples, the meaning of localization length for finite samples is not clear. Below, we follow the approach developed in the theory of quasi-1D disordered solids which is based on the evaluation of multifractal localization lengths (see, e.g., [15]). One of the commonly used quantities in this approach is the so-called entropic localization length, defined through the information entropy $\mathcal{H}_N$ of eigenstates:

$$\mathcal{H}_N = -\sum_{n=1}^{N} w_n \ln w_n \qquad w_n = |\varphi_n^2| \tag{3}$$

where φ_n is the nth component of an eigenstate in a given finite basis. For eigenstates normalized as $\sum_n w_n = 1$, the simplest case of $\varphi_n = N^{-1/2}$ results in the entropy being equal to the maximum value, namely $\mathcal{H}_N = \ln(N)$. We define therefore the localization length l_N as the number of basis states occupied by the eigenstate φ_n; the latter is equal to $\exp(\mathcal{H}_N)$. Similar definitions have been used for the first time in [18] where different characteristics of eigenstates have been discussed in solid-state applications. One can see that in the other limiting case of an exponentially localized state with $\varphi_n = l_\infty^{-1/2} \exp(-|n - n_0|/l_\infty)$, the quantity l_N is proportional to l_∞; in fact $l_N \approx e l_\infty$ (assuming $l_\infty \ll N$). One should note that, in general, the amplitudes φ_n fluctuate strongly with n and thus the coefficient of proportionality between l_N and l_∞ depends on the type of fluctuation.

In order to study for the quasi-1D solids the properties of chaotic states, localized on some scale in the finite basis it was suggested in [13, 14] that one could normalize the localization length l_N in such a way that in the extreme case of fully extended states the quantity l_N would be equal to the size of the basis N. In such an approach, the entropic localization length $l_N^{(1)}$ is defined as

$$l_N^{(1)} = N \exp(\langle \mathcal{H}_N \rangle - \mathcal{H}_{ref}). \tag{4}$$

In equation (4) the ensemble averaging $\langle \ldots \rangle$ is performed over the number of eigenstates with the same structure. The normalization factor $\mathcal{H}_{ref}$ has the meaning of an average entropy of the completely extended random eigenstates in the finite basis; therefore, it can be easily found analytically [13]:

$$\mathcal{H}_{ref} = \psi\left(\frac{N}{2} + 1\right) - \psi\left(\frac{3}{2}\right) \approx \ln\left(\frac{N}{2.07}\right) \tag{5}$$

where ψ is the digamma function and the distribution of components φ_n is assumed to correspond to the gaussian orthogonal ensemble (GOE). From equation (5) one can see that for $N \gg 1$ the entropic localization length of random eigenstates, defined simply as $\exp(\mathcal{H}_{ref})$, is approximately 2.07 times less than N; this result is due to gaussian fluctuations in the components φ_n.

Analogously, the whole set of localization lengths $l_N^{(q)}$ can be defined in the following way [15, 19]:

$$l_N^{(q)} = N \left(\frac{\langle P_q \rangle}{P_{ref}^{(q)}}\right)^{1/(1-q)} \qquad q \geqslant 2 \tag{6}$$

where

$$P_q = \sum_{n=1}^{N} (w_n)^q \tag{7}$$

and $P_{ref}^{(q)}$ is the average value of P_q for the reference ensemble of completely extended states. One should note that for the particular case $q = 2$ the quantity P_2 is known as

the participation ratio; it is widely used in solid-state physics. In the limiting case of the GOE, one can find that $P_{ref}^{(2)} = 3/N$; therefore, the inverse participation ratio $(P_2)^{-1}$, which is commonly taken as the definition of localization length, for random eigenstates is three times less than the 'actual' length N.

In fact, the above expressions for the localization lengths $l_N^{(q)}$ are defined through the $2q$th moments of a distribution of components φ_n of eigenstates—not normalized to $P_{ref}^{(q)}$, the quantities given by equation (6) are well known in the multifractal analysis of wave functions. Such normalization turns out to be extremely important when establishing scaling properties of eigenfunctions. Indeed, by normalizing the localization lengths $l_N^{(q)}$ to the size N of the sample

$$\beta_q = \frac{l_N^{(q)}}{N} \tag{8}$$

one can expect, in the spirit of renormalization theory, that the set of dimensionless parameters β_q is the proper entity to characterize generic properties of eigenstates for finite samples. According to the scaling conjecture in the modern theory of disordered solids, it was assumed [20] that for quasi-1D disordered models described by band random matrices each quantity β_q depends on the scaling parameter λ only, which is the ratio of the localization length l_∞ for the infinite sample to the size N of the sample itself. Therefore, the scaling relation can be written as

$$\beta_q = f_q(\lambda) \qquad \lambda = \frac{l_\infty.}{N} \tag{9}$$

Detailed studies, both numerical and analytical, have confirmed this conjecture for different models like the kicked rotator model and band random matrices (see, e.g., [14, 15, 16] and references therein). Moreover, the scaling function $f_q(\lambda)$ has also been found.

Our main question is whether a relation of the type of equation (9) is also valid for our dimer model with the correlated disorder. The first nontrivial question arises about the reference ensemble for the computation of the average entropy $\mathcal{H}_{ref}$. Indeed, in application to 1D Anderson-type models (see details in [21]), the reference ensemble cannot be chosen as an ensemble of full random matrices, like the GOE. This point is related to the fact that in the Anderson case fully extended states are not gaussian random functions but just plane waves which arise for zero disorder. In the dimer model, the situation is even more complicated due to strong dependence of the localization length on the energy. However, and this is our expectation, in spite of the presence of the extended states at the critical energies, scaling properties of the eigenstates in the dimer model of finite size N are of the generic type discovered for 1D and quasi-1D disordered models.

For this reason and in the spirit of [21, 22], we define the normalization factors $\mathcal{H}_{ref}$ and $P_{ref}^{(q)}$ from the solution of equation (2) for zero disorder, $\epsilon_n = 0$:

$$E^k = 2\cos\frac{k\pi}{N+1} \tag{10}$$

$$\varphi_n^k = \sqrt{\left(\frac{2}{N+1}\right)}\sin\frac{nk\pi}{N+1} \tag{11}$$

with $k, n = 1, \ldots, N$. The entropy $\mathcal{H}_{ref}$ of the above eigenfunctions in the large-N limit has the same value for every eigenvalue E^k, i.e.

$$\mathcal{H}_{ref} = \ln(2N) - 1 \tag{12}$$

and, correspondingly,

$$P_{ref}^{(2)} = \frac{3}{2N}. \tag{13}$$

3. Scaling properties of localization lengths in the centres of energy bands

Since all results depend on the difference $\epsilon_{A,B} = |\epsilon_A - \epsilon_B|$ but not on the actual values of ϵ_A and ϵ_B separately, we can set $\epsilon_A = 0$ for simplicity. One should stress that both localization lengths l_∞ and $l_N^{(q)}$ are functions of the energy E. For this reason, in our numerical experiments we consider ensembles of states specified by the values of the energy E in a small window ΔE and by different realizations of random on-site energies ϵ_n. We choose the size of the energy window in such a way that for every chosen value of ϵ_B the localization length l_∞ is approximately constant inside this window (in all the cases $\Delta l_\infty / l_\infty \leqslant 0.06$).

In order to study scaling properties of the localized eigenstates we have used the transfer matrix method for infinite chains as well as the direct diagonalization of the Hamiltonians that one associated with equation (1), for finite chains of size N. To find the localization length l_∞ we have studied the asymptotic behaviour of the random matrix product $\prod \mathbf{M}_n$, where $\mathbf{M}_n$ is defined through the relation

$$\boldsymbol{\xi}_{n+1} = \mathbf{M}_n \boldsymbol{\xi}_n \qquad \mathbf{M}_n = \begin{pmatrix} v_n & -1 \\ 1 & 0 \end{pmatrix} \qquad v_n = E - \epsilon_n \tag{14}$$

for the vector $\boldsymbol{\xi}_n = (x_n, x_{n-1})$ with the matrix $\mathbf{M}_n$ known as the transfer matrix. Then the localization length l_∞ is the inverse Lyapunov exponent γ; the latter is evaluated as the exponential decay rate of an initial vector ξ_1:

$$l_\infty^{-1} = \gamma = \lim_{N\to\infty} \frac{1}{N} \left(\ln \prod_{n=1}^{N} |\mathbf{M}_n \boldsymbol{\xi}_n| \,\big/ |\boldsymbol{\xi}_1| \right). \tag{15}$$

Although the Lyapunov exponent γ for finite N depends on a particular realization of the disorder, for $N \to \infty$ it converges to its mean value. For the above calculations we have used samples of length 5×10^5 for relatively large values of ϵ_B and up to 4×10^6 for small values of ϵ_B.

To reveal scaling properties of the localization length for finite samples, we have computed two localization lengths $l_N^{(1)}$ and $l_N^{(2)}$ according to the relations discussed in the previous section, with the normalization factors $\mathcal{H}_{ref}$ and $P_{ref}^{(2)}$ given by equation (12) and equation (13). In the computations of these lengths, the energy window was taken in the centre of the spectrum, around the value $E = \epsilon_B/2$ for ϵ_B equal to 2, 1.8, 1.6, 1.2, 1, 0.8, 0.6, 0.4, 0.35 and for the fixed value of N. The widths of the windows were numerically chosen to provide a small change of localization length inside any of the windows. The values of β_1 and β_2 were obtained by the averaging over an ensemble of random samples of size $N = 100$–800 for the values of ϵ_B cited previously. As a result, the total numbers of eigenstates in the energy windows were more than 1000.

All of the data were fitted to the scaling function β_q found for quasi-1D disordered models [15]:

$$\beta_q = \frac{c_q \lambda}{1 + c_q \lambda}. \tag{16}$$

In fact, this scaling relation is exact only for $q = 2$; however, for other cases of small values of q, including $q = 1$, it is very close to the correct one (see details in [15]).

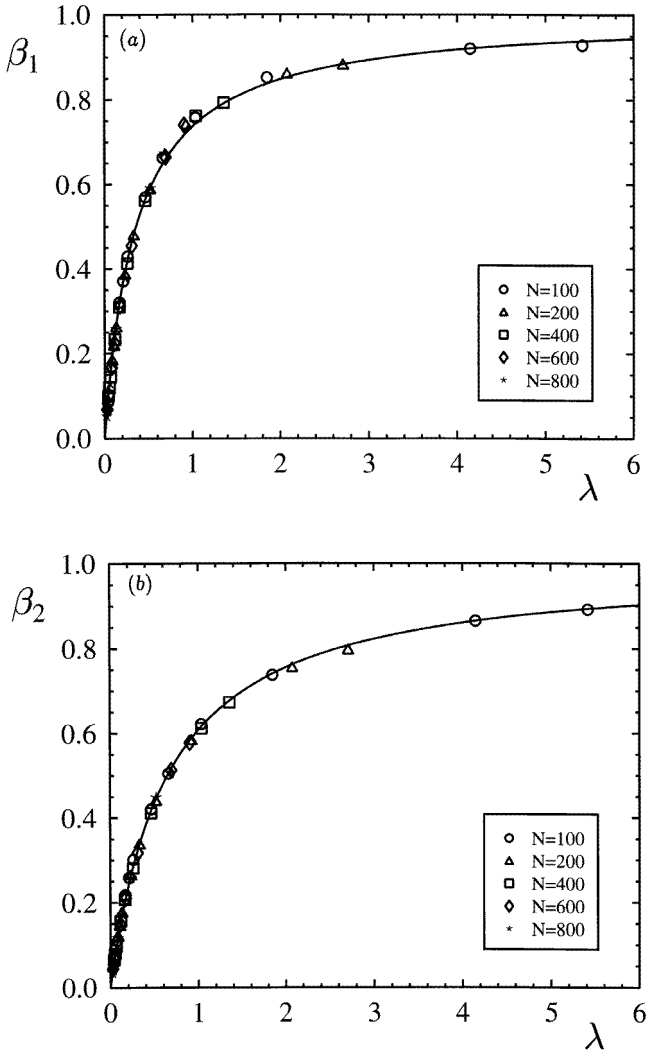

Figure 1. Scaling of β_q as a function of the localization ratio $\lambda = l_\infty/N$ for the RDM with $Q = 0.5$ and $\epsilon_A = 0$. The energies E are taken in an energy window centred at $E = \epsilon_B/2$ for values of $\epsilon_B = 2.0$; 1.8; 1.6; 1.2; 1.0; 0.8; 0.6; 0.4; 0.35. Smooth curves correspond to the dependence (16) with c_q as a fitting parameter. (a) Scaling for β_1 with $c_1 = 2.80$. (b) Scaling for β_2 with $c_2 = 1.55$.

Numerical data reported in figure 1 give clear evidence of a scaling of the type described by equation (16). The fitting parameters c_q are equal to $c_1 = 2.80$ and $c_2 = 1.55$. From this figure one can see that the behaviour of β_q is very different in the two limits of very localized ($\beta_q \ll 1$) and extended ($\beta_q \approx 1$) eigenstates. The dependence of equation (16) has the remarkable property which can be seen in other variables:

$$Y_q = \ln\left(\frac{\beta_q}{1-\beta_q}\right) \qquad X = \ln\left(\frac{l_\infty}{N}\right) \tag{17}$$

which are more convenient when considering the whole region of both very localized and

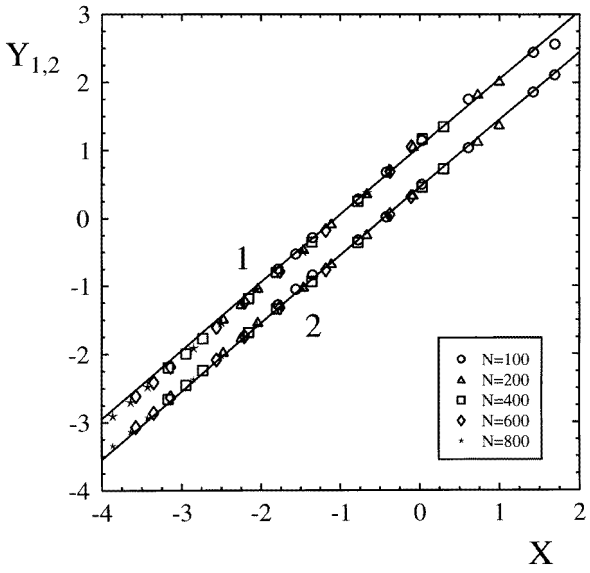

Figure 2. The scaling of β_1, β_2 as a function of $\lambda = l_\infty/N$ in the variables $Y_{1,2}$ and X (see (17)) for the same values of the parameters as in figure 1. Straight lines (1) and (2) correspond to expression (18) with $a_1 = 1.05$; $b_1 = 1$ and $a_2 = 0.45$; $b_2 = 1$ respectively.

extended states. Indeed, in these variables the scaling has an extremely simple form:

$$Y_q = a_q + b_q X \tag{18}$$

with $b_q = 1$ and $a_q = \ln(c_q)$. The data for the scaling in variables Y, X are presented in figure 2. The fitting parameters $b_{1,2}$ are found to be quite close to 1, i.e. $b_1 = 1.02$ and $b_2 = 0.98$; for this reason in figure 2 we put $b_1 = b_2 = 1$. The remarkable result is that the above simple scaling relation holds over a very large region of the scaling parameter $\lambda = l_\infty/N$. According to the fit to the dependence of equation (18), the values $a_{1,2}$ are $a_1 = 1.05$ and $a_2 = 0.45$, which gives $\Delta a_{1,2} = a_1 - a_2 = 0.6$. It is very interesting that these values of $a_{1,2}$ are the same as for the common Anderson model [22] in the centres of energy bands. This fact is very important in establishing the link between the RDM and Anderson models of finite size.

It is of special interest to relate the entropy localization length $l_N^{(1)}$ and the localization length $l_N^{(2)}$ associated with the inverse participation ratio. Their interdependence is shown in figure 3. We see that they are approximately equal for very localized and very extended states. It is also clear that β_2 is always less than β_1 since $P_N^{(q)} < P_N^{(q+1)}$, due to the definition of equation (7). Using the definition of equation (16) one can find the relation between β_1 and β_2:

$$\beta_2 = \frac{c\beta_1}{1 + (c-1)\beta_1} \qquad c = \frac{c_2}{c_1}. \tag{19}$$

4. Scaling of localization lengths near the critical energies

In the previous section we have shown that the scaling law of equation (16), found for fully disordered 1D and quasi-1D models, also holds in our dimer model of finite size when considering eigenstates in the centres of energy bands. In a sense, this property may

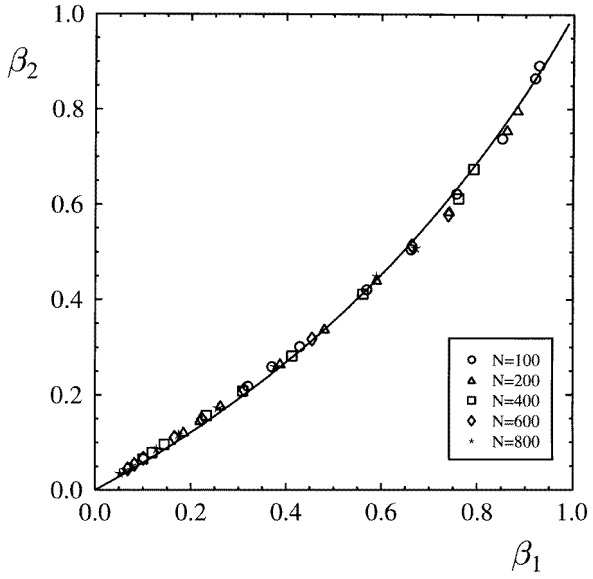

Figure 3. A plot of β_2 as a function of β_1. It is interesting to note that the fitting curve has the same form (19) as the ones for the case of $\beta_{1,2}$ plotted in figure 1 as a function of $\lambda = l_\infty/N$.

be expected, since far from critical energies where the localization length diverges, the eigenstates are assumed to be similar to that known for disordered models. The important question is whether or not this scaling holds for all energies inside the band—in particular, near the critical energies $E_{cr} = \epsilon_A, \epsilon_B$. Direct numerical computation of the localization length l_∞ through the transfer matrix method is very difficult in this energy region due to very weak convergence of Lyapunov exponents. For this reason, we have used the analytical expression which was derived for l_∞ near the critical energies in an approach developed in [8]:

$$l_\infty(E) \approx \frac{2\sin^2\mu_0}{Q\delta^2\cos^2\mu_0} \qquad 2\cos\mu_0 = E. \tag{20}$$

Here, the factor Q stands for the probability for the pair $\epsilon_n = \epsilon_{n+1} = \epsilon_B$ to appear, and δ is defined by the relation $E = \epsilon_B - \delta \approx \epsilon_B$ [23]. We recall that in our case $Q = 1/2$ and $\epsilon_A = 0$ have been assumed for simplicity. From equation (20) one can find that if the value of ϵ_B is far from the stability border $E_B = 2$, and the distance $\Delta = 2 - \epsilon_B$ is large compared to $\delta = \epsilon_B - E$, the localization length diverges as

$$l_\infty \approx \frac{2\Delta}{Q\delta^2} \qquad \delta \ll \Delta \ll 1. \tag{21}$$

In the other limit case of $\epsilon_b = 2$ we have

$$l_\infty \approx \frac{2}{Q\delta} \qquad \delta \ll 1,\ \Delta = 0. \tag{22}$$

It is interesting to note that the same expressions, equations (21) and (22), are obtained in [17] by assuming that localization length l_∞ is determined by the reflection coefficient from a single pair $\epsilon_n = \epsilon_{n+1}$, embedded in a perfect lead. It is of interest to check how accurate estimates found in [8] and [17] are, and if one can apply them for any energy inside the band.

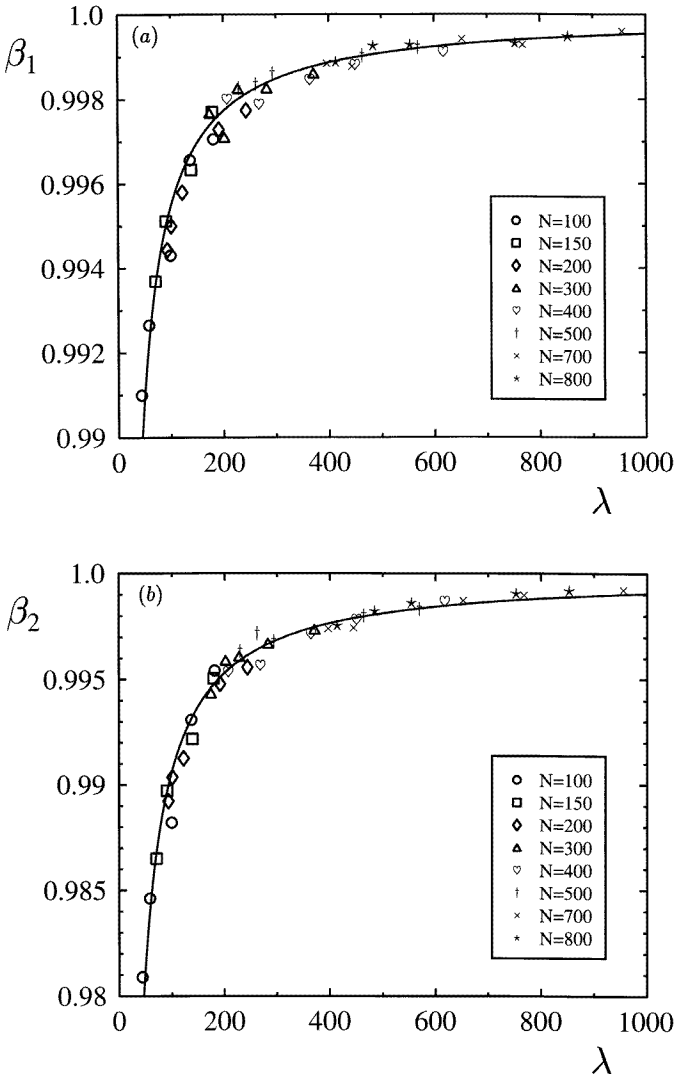

Figure 4. As in figure 1, but for energies close to the critical one ($E_{cr} = \epsilon_B$), for $q = 1$ (a) and $q = 2$ (b). The values of ϵ_B are taken as $\epsilon_B = 1.8$; 1.6; 1 .4; 1.2; 1.0.

To find the localization lengths $l_N^{(1)}$ and $l_N^{(2)}$ for finite samples of size N, we have used the same approach as described in the previous section, by examining the eigenstates with energies in a small energy window $\Delta E \leqslant 10^{-2} E_{cr}$ near the critical energy $E_{cr} = \epsilon_B$. Yet, since, in the region of critical energies, $l_\infty(E)$ and thus also the localization properties of eigenstates depend on the energy in a singular way (see equation (21) and equation (22)), we took from the energy window only the eigenvector with the corresponding eigenstate which is closer to E_{cr} (but always different from it, $E \neq E_{cr}$). This was a natural choice for studying statistical properties of eigenstates with similar localization properties (i.e. just the eigenstates near the totally extended one). The average values of $l_N^{(q)}$ were obtained by statistically averaging over an ensemble of more than 3000 samples of size $N = 100$–800

with different pair-correlated disorder. The results are reported in figure 4 together with a fit to equation (16). One can see a quite good scaling of the form of equation (16), in spite of fluctuations which are much larger in this energy region compared to those in the centres of bands. The fitting coefficients $c_1 = 2.20$ and $c_2 = 1.06$ are slightly less than those in the band centres. This fact may be explained by the approximate character of the analytical expression in equation (20) (one should also note that for the values of β_q very close to the limit $\beta_q = 1$ the computational errors are very large).

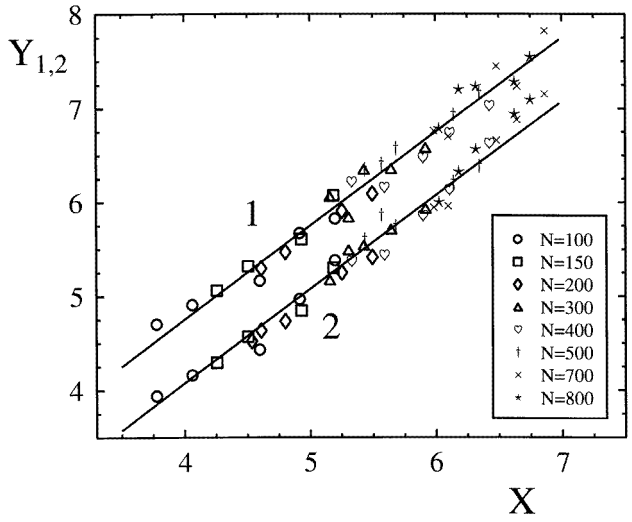

Figure 5. As figure 2, but for the parameters of figure 4. Straight lines (1) and (2) correspond to expression (18) with $a_1 = 0.75$; $b_1 = 1$ and $a_2 = 0.08$; $b_2 = 1$ respectively.

In figure 5 the same data are represented in the variables of equation (17), with the fit corresponding to the dependence given by equation (18). It is interesting that in spite of a slight difference for the coefficients $a_1 = 0.75$ and $a_2 = 0.08$ in comparison to those found in the centres of bands, the shift $\Delta a_{1,2} = a_1 - a_2 = 0.67$ remains almost the same (compared to 0.6).

5. Summary

We have studied a 1D tight-binding model with binary on-site disorder that is randomly assigned at every pair of sites. For such a model we know that there exist two special energies E_{cr} at which transparent states appear [3, 6, 7, 8]. For other energies—but close to critical ones—the localization length is very large, leading to nearly transparent states that are of great importance in the conducting properties of finite samples. This property is quite different from that of genuine disordered models of Anderson type.

Our numerical study of random dimer models of finite size deals with the scaling properties of the eigenstates. This study was motivated by the remarkable scaling law that has been found for different 1D and quasi-1D models, both dynamical (the kicked rotator on a torus [13, 14]) and disordered (1D Anderson and Lloyd models [21, 22] and quasi-1D models [15, 20]). These latter results indicate that eigenstates in finite samples with disorder have generic properties, regardless of the details of the disorder.

The main result of our computations is that scaling properties of eigenstates of finite dimers are of the same type as for the disordered models mentioned above in spite of the existence of nearly transparent states. In particular, both the entropy localization length and the localization length from the inverse participation ratio normalized in the proper way follow the universal scaling law of equation (16).

The scaling relation of equation (16) can be also represented in a very intriguing form [21, 22, 24]:

$$\frac{1}{l_N^{(q)}(\epsilon, E)} = \frac{1}{l_\infty^{(q)}(\epsilon, E)} + \frac{1}{l_N^{(q)}(0, E)} \tag{23}$$

which still has no physical basis. In equation (23), $l_N^{(q)}(\epsilon, E)$ represents the localization length for a finite sample and finite disorder, $l_\infty^{(q)}(\epsilon, E)$ is the localization length for an infinite sample with the same disorder and $l_N^{(q)}(0, E)$ is the localization length for a finite sample with zero disorder. One should stress that all three localization lengths are defined in the same way, given through expressions of equations (6) and (7). One can see that the form of equation (23) is parameter independent; the same form holds also for the 1D Anderson and Lloyd models (see [21, 22]).

In our numerical study the energy window has been chosen in the middle of the spectrum as well as close to the critical energy, giving the same scaling form—equation (16). The slight difference in the coefficients c_q for these two energy regions seems to indicate that the analytical expression equation (20) needs some correction related to an additional dependence on the energy when the latter is not close enough to the critical one. Our results indicate that the same scaling is expected to hold for other values of the energy inside the band. One should note that the scaling given by equation (16) (or, equally, equation (23)) can be used to check the accuracy of expressions for the localization length l_∞ as regards the dependence of the parameters E and ϵ_b, if for some values of these parameters the scaling function β_q is found with a high accuracy.

It is of interest to check the scaling behaviour of localization lengths corresponding to the higher moments $q \geqslant 3$ in equation (6). Analytical treatments [15] for disordered models have shown that the scaling law given by equation (16) holds approximately for higher moments also. The correct expression for $\beta_q(\lambda)$ is known only in the limit case of very localized ($\lambda \ll 1$) and extended ($\lambda \gg 1$) states. It has the same form as equation (18) with $b_q = 1$ but with different values of a_q in these limits (see details in [15]). On the basis of our results for $q = 1, 2$, it is quite natural to expect that for the dimer model the correspondence to the analytical predictions [15] should also hold for higher moments; however, this question remains open. One should note that fluctuations of localization lengths l_q increase very substantially with increasing q; this leads to serious computational problems.

Finally, we should comment that the results obtained in the present work can be generalized to cases with correlation blocks larger than dimers, namely m-blocks with $m = 3, 4, 5, \ldots$. In these more general cases the following surprising result holds: given an arbitrary distribution of correlated blocks with *even* length, i.e. an arbitrary distribution of dimers, tetramers, hexamers, octamers, etc, with the same energy ϵ, that populate a lattice with sites that have some other energy value, there is always a resonant energy $E_{cr} = \epsilon$ that corresponds to a delocalized state. This result can be easily deduced from the general expressions of [8]. On physical grounds, we expect the localization properties of the eigenstates of this system to follow similar scaling laws to the ones derived in the present work.

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